

MATHEMATICS

Calculation Strategies
(Multiplication and Division)

AIMS OF THE SESSION:

- To learn about the structural laws underpinning numerical concepts
- To understand the algorithms for multiplication and division exemplified in the NC and how they could be used and modelled
- To understand how calculation strategies develop and progress
- To understand what children's common errors and misconceptions are in the four operations
- To be aware of what remedial action can be planned to help children
- ▶ To be aware of are the common difficulties children experience in written calculations and how can these be overcome

REMINDE YOURSELF...

- ▶ What is the law?
- ▶ Can you provide an example?
- ▶ What are the misconceptions surrounding the law?

Commutative
law

Associative
law

Distributive law

WHAT IS AN ALGORITHM?

- ▶ A process or set of rules to be followed in calculations or other problem-solving operations, especially by a computer
- ▶ They **MUST** be followed in the correct order to achieve the desired outcome

But are our pupils secure the background and understanding behind the algorithm?
Or are they just following a set of instructions?

COMMON ERRORS AND MISCONCEPTIONS WITH MULTIPLICATION AND DIVISION



- Mistakes with recall of multiplication tables and other basic facts
- Not secure in place value and therefore demonstrates a lack of understanding of the concept
- Lack of understanding of relevant vocabulary
- Over-generalisation: hence the importance of real understanding and not just mechanical learning of a procedure
- Unable to make connections between different methods

The easiest way to identify and address the misconception...

Ask the child to explain how they calculated.

COMMON ERRORS AND MISCONCEPTIONS

How can some of these misconceptions be overcome?

- Be aware of them – model as a ‘mistake’, discuss them, use a teaching points
- Model alongside a familiar method from their previous learning so they can make connections
- Support with the use of mathematical resources so they can understand why, rather than just ‘do as the teacher says’

YOUR TASK...

To plan a number of mini-tasks to determine whether a Y4 child truly understands multiplication and division methods

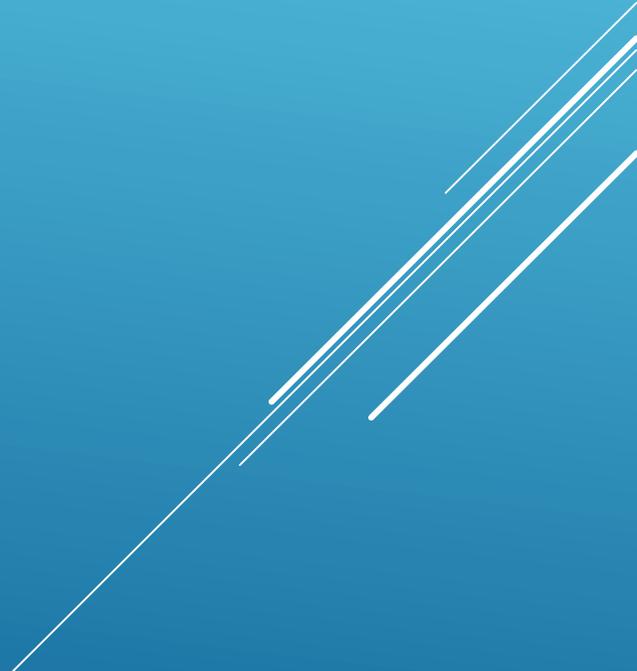
You've been presented with a piece of work with 24 correctly answered calculations – how do you know the child has truly understood the methods involved in multiplication and division?

- ▶ Using dienes/place value counters to model and reason alongside using the method
- ▶ Using mistakes in workings as teaching points – can they explain why the answer is correct/incorrect? Can they use an estimate to check the answer?
- ▶ Verbally explaining using mathematical vocabulary
- ▶ Use of missing numbers and justifying why the gaps should be a certain number
- ▶ Can they represent the calculation as a visual model?
- ▶ Use of the mastery activities
- ▶ Including zeros within the calculation, to assess understanding
- ▶ Comparing two different methods solving the same calculation

EXPECTATIONS OF Y4

Year	Number and place value	Addition and subtraction	Multiplication and division	Fractions	Measurement	Geometry		Statistics
						Properties of shape	Position and direction	
YEAR 4	<p>Pupils should be taught to</p> <ul style="list-style-type: none"> count in multiples of 6, 7, 9, 25 and 1000 find 1000 more or less than a given number count backwards through zero to include negative numbers recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones) order and compare numbers beyond 1000 identify, represent and estimate numbers using different representations round any number to the nearest 10, 100 or 1000 solve number and practical problems that involve all of the above and with increasingly large positive numbers read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value. 	<p>Pupils should be taught to:</p> <ul style="list-style-type: none"> add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate estimate and use inverse operations to check answers to a calculation solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why. 	<p>Pupils should be taught to:</p> <ul style="list-style-type: none"> recall multiplication and division facts for multiplication tables up to 12×12 use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers recognise and use factor pairs and commutativity in mental calculations multiply two-digit and three-digit numbers by a one-digit number using formal written layout solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects. 	<p>Pupils should be taught to:</p> <ul style="list-style-type: none"> recognise and show, using diagrams, families of common equivalent fractions count up and down in hundredths; recognise that hundredths arise when dividing an object by a hundred and dividing tenths by ten. solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number add and subtract fractions with the same denominator recognise and write decimal equivalents of any number of tenths or hundredths recognise and write decimal equivalents to $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$ find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths round decimals with one decimal place to the nearest whole number compare numbers with the same number of decimal places up to two decimal places solve simple measure and money problems involving fractions and decimals to two decimal places. 	<p>Pupils should be taught to:</p> <ul style="list-style-type: none"> Convert between different units of measure (e.g. kilometre to metre; hour to minute) measure and calculate the perimeter of a rectilinear figure (including squares) in centimetres and metres find the area of rectilinear shapes by counting squares estimate, compare and calculate different measures, including money in pounds and pence read, write and convert time between analogue and digital 12 and 24-hour clocks solve problems involving converting from hours to minutes; minutes to seconds; years to months; weeks to days. 	<p>Pupils should be taught to:</p> <ul style="list-style-type: none"> compare and classify geometric shapes, including quadrilaterals and triangles, based on their properties and sizes identify acute and obtuse angles and compare and order angles up to two right angles by size identify lines of symmetry in 2-D shapes presented in different orientations complete a simple symmetric figure with respect to a specific line of symmetry. 	<p>Pupils should be taught to:</p> <ul style="list-style-type: none"> describe positions on a 2-D grid as coordinates in the first quadrant describe movements between positions as translations of a given unit to the left/right and up/down plot specified points and draw sides to complete a given polygon. 	<p>Pupils should be taught to:</p> <ul style="list-style-type: none"> interpret and present discrete and continuous data using appropriate graphical methods, including bar charts and time graphs solve comparison, sum and difference problems using information presented in bar charts, pictograms, tables and other graphs.

UNDERTAKE ACTIVITIES WITH Y4 CHILD

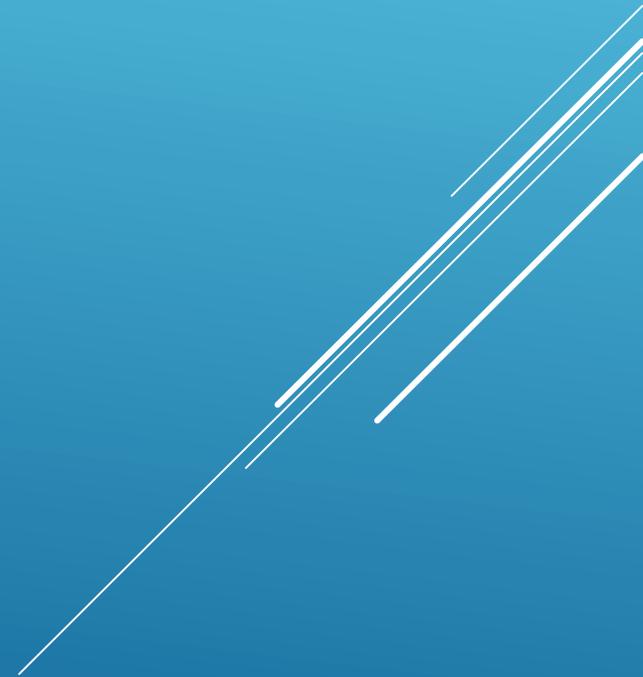


DISCUSSION TIME WITH PARTNER

- ▶ Is the child working at ARE (Age Related Expectation)?
Are they secure at year 4? How do you know?
- ▶ Did they have any misconceptions which prevented them from progressing and achieving? How did you address these misconceptions?
- ▶ What would you suggest are the next steps for this child to make progress?



BREAK



USING YOUR WORK SAMPLES...

- ▶ Which visual models are evident in your books to support the understanding of multiplication and division?

Multiplication

- ▶ Year 3: multiply 2-digit numbers by a single digit number.
- ▶ Year 4: Multiply 2 and 3 digits by a single digit
- ▶ Year 5: Multiply up to 4 digits by up to 2 digits.
- ▶ Year 6: Multiply decimals with up to 2 decimal places by a single digit.

Number line?
Grid method?
Expanded column method?
Short column method?

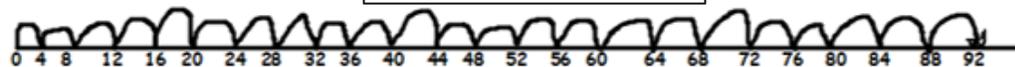
Division

- ▶ Year 3: Divide 2-digit numbers by a single digit
- ▶ Year 4: Divide up to 3-digit numbers by a single digit.
- ▶ Year 5: Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.
- ▶ Year 6: Divide at least 4-digit numbers by single and 2-digit numbers (including decimals).

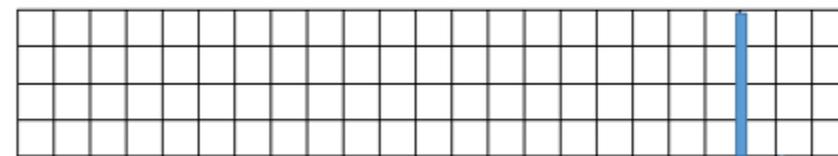
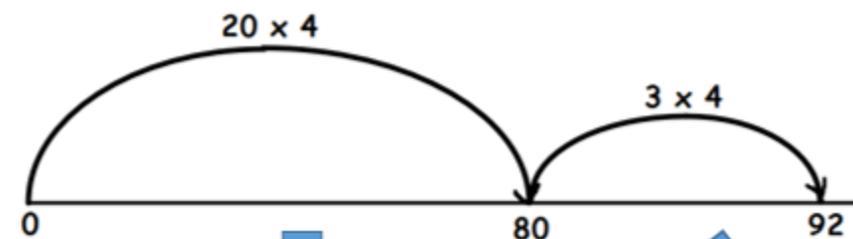
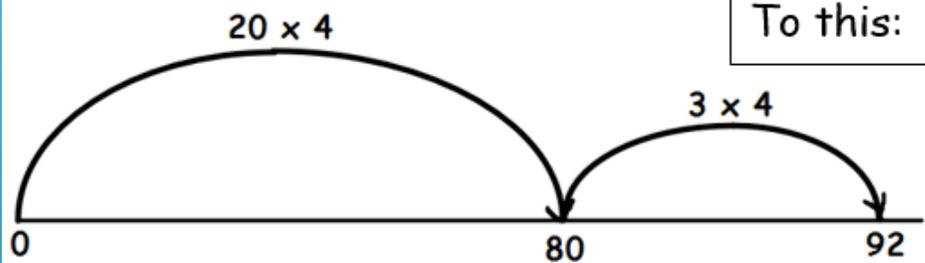
Number line?
Chunking using column method?
Compact method (bus stop)?

$$23 \times 4 = 92$$

From this:



To this:



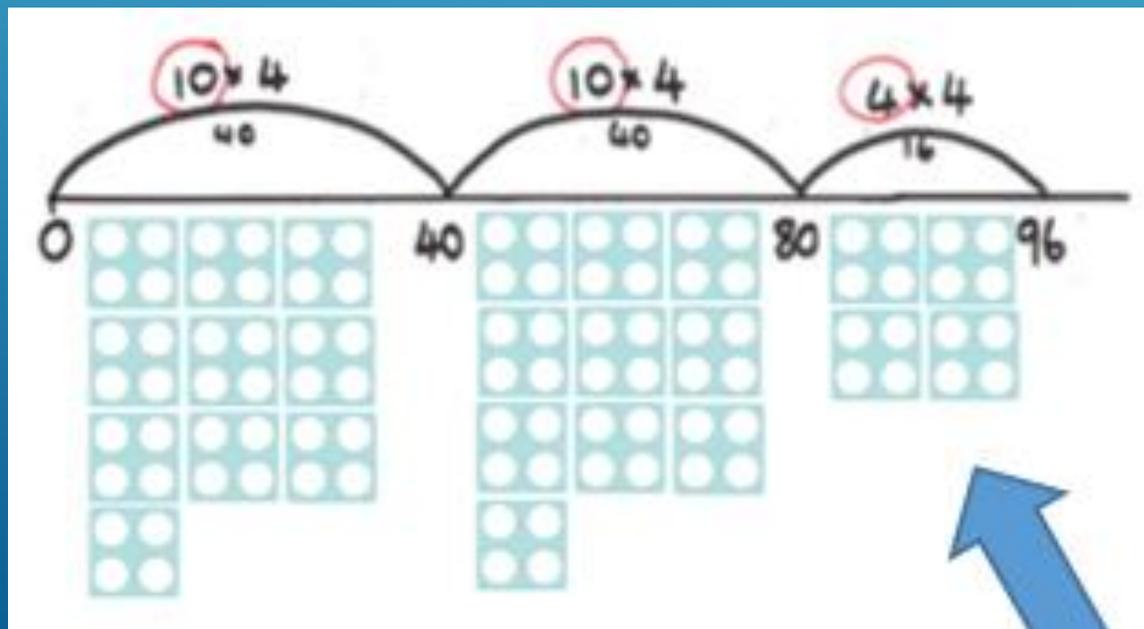
20 x 4

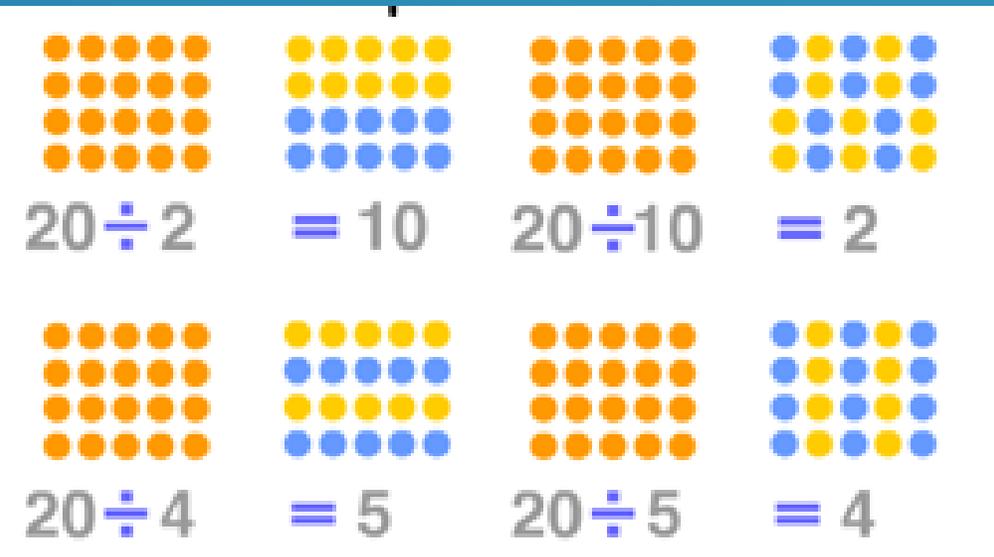
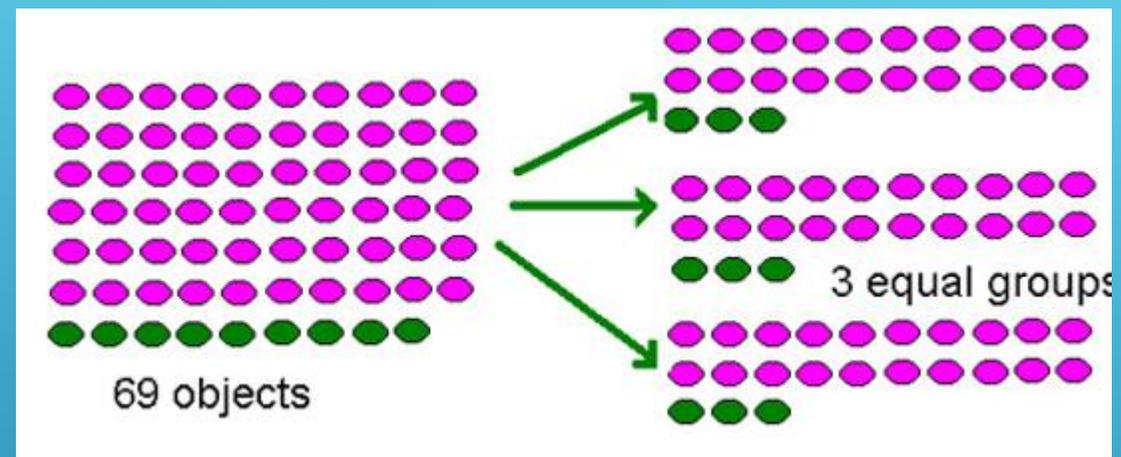
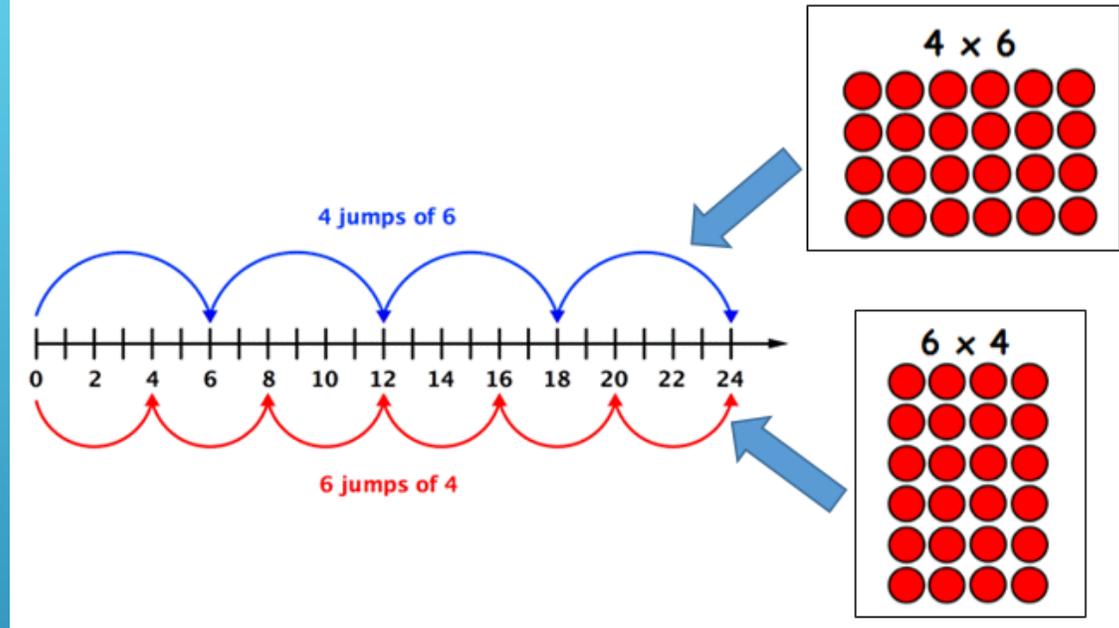
3 x 4



20 x 4

3 x 4





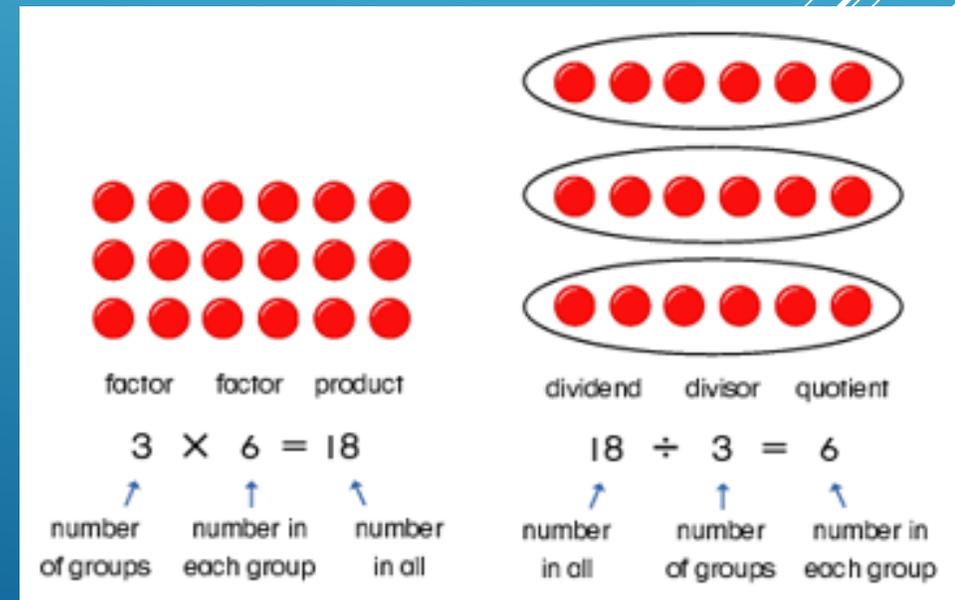
$3 \times 5 = 15$

So...

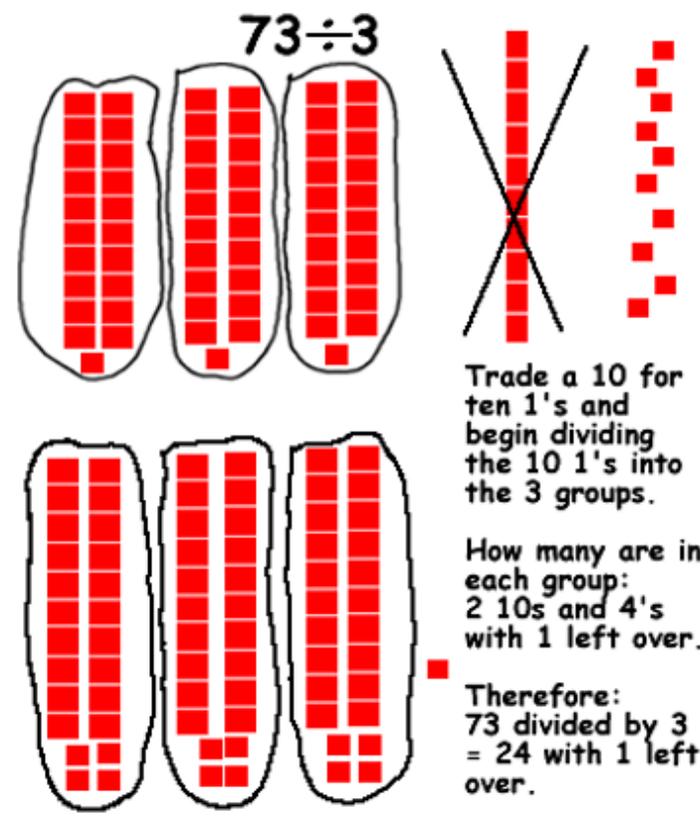
$30 \times 5 = 150$

$3 \times 50 = 150$

$30 \times 50 = 1500$



	6	7	7	\times	9	
\times	6	0	0		7	0
9	5	4	0	0	6	3

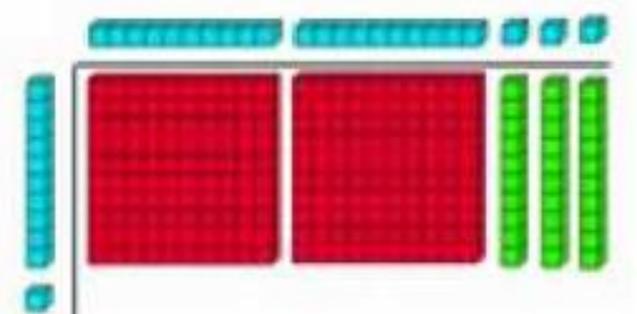


$26 \div 3$

Represented physically: sharing or grouping strategy?



23×11

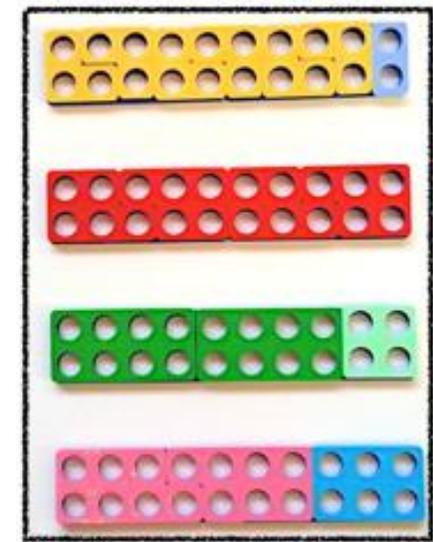


$20 \div 3 = 6r2$

$20 \div 5 = 4$

$20 \div 8 = 2r4$

$20 \div 7 = 2r6$



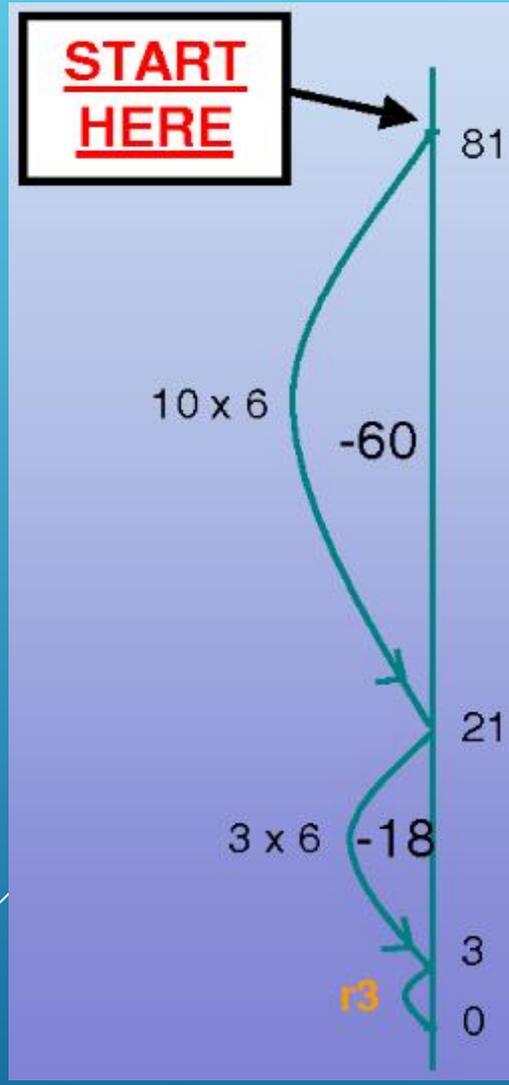
$$\begin{array}{r}
 234 \\
 \times 312 \\
 \hline
 468 \\
 2340 \\
 70200 \\
 \hline
 73008
 \end{array}$$

$$\begin{array}{r}
 96 \\
 \times 32 \\
 \hline
 192 \\
 2880 \\
 \hline
 3072
 \end{array}$$

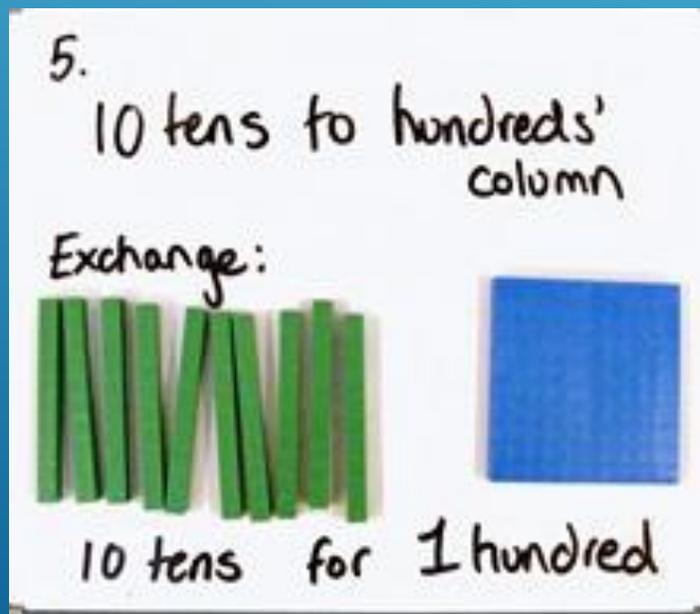
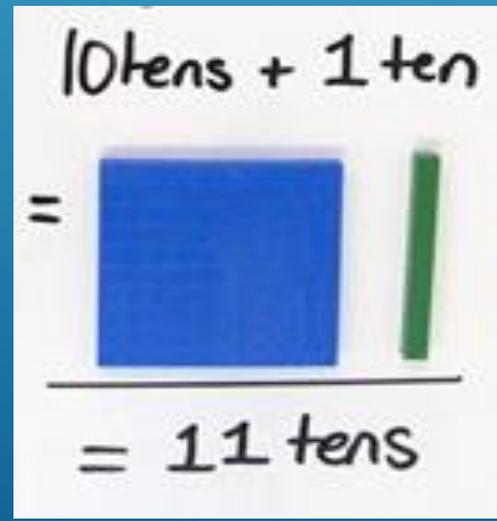
this is 96×2
 this is 96×30
 this is 96×32

$$\begin{array}{r}
 43 \\
 \times 21 \\
 \hline
 430 \\
 86 \\
 \hline
 903
 \end{array}$$

Multiplying by a tens digit...
 ...so need a 0 here



$$\begin{array}{r}
 32 \\
 \times 24 \\
 \hline
 8 \quad (4 \times 2) \\
 120 \quad (4 \times 30) \\
 40 \quad (20 \times 2) \\
 600 \quad (20 \times 30) \\
 \hline
 768
 \end{array}$$



POST TASK:



Please use your experience from the last two training sessions which have involved you probing deeper into a child's learning and understanding in mathematical calculations.

Liaise with your CTM to decide on a child who is more difficult to accurately assess. This may be due to a number of factors:

- A lack of written evidence in books?
- Reluctance to verbalise their understanding within the class?
- Heavily reliant on other children in the class?
- Producing correct answers when calculating, but unable to apply learning or make connections?
- Able to follow a calculation procedure, but lack the understanding to support the process?

Use a variety of activities, such as the activities you have used in the training sessions, to unpick the child's learning and understanding, to allow you to form a more accurate assessment of the child.

Please use this evidence (work samples, photos, planning) to build your portfolio of evidence for TS6 in your PDP.